

VALIDITY OF THE ESTIMATE OF THE CURRENT DENSITY ALONG CLUSTER ORBIT WITH SIMULATED MAGNETIC DATA

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ABSTRACT

The Tsyganenko magnetic field model is used to simulate magnetic field measurements to be carried out aboard the 4 Cluster spacecraft. The barycentric and contour integral methods are used to estimate the current density along Cluster trajectory. Due to the non-homogeneity of the current density profile associated with the Tsyganenko model, the estimated values differ from the exact values. We investigate the effect upon this error of the finite distances between the spacecraft and of the shape of the tetrahedron formed by the 4 spacecraft. Comparison between estimated and exact values is used to test the usefulness of quality criteria based upon purely geometric considerations.

1. INTRODUCTION

Data from the fluxgate magnetometers embarked onboard the 4 Cluster spacecraft can be used to estimate vectorial parameters deduced from the magnetic field, such as $\text{curl}\mathbf{B}$, $\text{div}\mathbf{B}$ and $\text{grad}\mathbf{B}$, for instance by using the barycentric coordinate method (Chanteur and Mottez Ref. 1). These parameters can be plotted as daily magnetic summaries such as those presented in companion paper (Coeur-Joly et al., Ref. 2). The examination of these daily summaries can help detecting crossings of current density structures, provided that the estimate of the current density structure is reliable. The errors on the estimate of vectorial parameters may be due to several factors, including the uncertainty in the measurement of the position of the spacecraft, the uncertainty on the measurement of magnetic fields, the validity of linear interpolation, and the effect of the shape of the Cluster tetrahedron, which can in certain cases amplify the total error (Robert and Roux., Ref. 3). It has been suggested (Robert and Roux, Ref. 4, Coeur-Joly et al., Ref. 2, Dunlop et al., Ref. 8) that $\text{div}\mathbf{B}$ could be used to estimate the error. Yet, the estimate of $\text{div}\mathbf{B}$ is subject to the same kind of limitation. It has been shown by Robert and Roux (Ref. 4) that the estimated value of $\text{div}\mathbf{B}$ cannot easily be related with the uncertainty deduced from the difference between the theoretical and the estimated value of the current density. The purpose of the present work is to try to characterize the quality of the estimate of the current density by using geometric quality factors in a situation where magnetic data along Cluster orbit are deduced from the Tsyganenko model. Then, the geometric quality factors can be plotted together with these parameters, and the quality of the estimate of the current density can be compared with the corresponding values of the geometric quality factors.

2. THE GEOMETRIC QUALITY FACTORS

Several geometric quality factors, computed from the position of each summit of the tetrahedron, have been defined in a preceding paper (Robert and Roux, Ref. 3). These quality factors have been compared with the accuracy of the estimate of the current density in a situation where the position of each spacecraft or the measurement of the magnetic field are known within a given accuracy. Let us briefly summarize here the method used and the results obtained.

2.1. The shape of the tetrahedron

Many configurations of the Cluster tetrahedron are selected in a big "reservoir" containing 4 kinds of configurations: (i) perfectly regular tetrahedra, (ii) "almost regular" tetrahedra, (iii) "almost planar" tetrahedra, and (iiii) "almost linear" tetrahedra. All tetrahedra have a finite volume, and the same characteristic size, i.e. the same mean inter spacecraft distance.

2.2. Geometric quality factors

For each tetrahedron, several geometric quality factors were computed from the position of each spacecraft with respect to the centre of gravity. All quality factors range between 0 and 1; the latter value corresponds to a perfectly regular tetrahedron and 0 corresponds to a degenerate configuration: flat, linear, or completely degenerate (all summits at the same location).

2.3. Crossing of current structure

Any model can be used for the current density, for instance a current tube with an homogeneous or a gaussian shaped current density. Characteristics parameters of the tube can be chosen: $R=5000$ or 10000 km for the radius, constant current density $J_0=10^{-8}$ Am⁻², a gaussian shaped mean square deviation $\sigma=R$. Characteristic (mean) distance between each Cluster spacecraft was $D_c=1000$ km.

2.4. Computation of the current density

When a given Cluster constellation crosses a current tube, moving along an arbitrary direction, the magnetometers record the corresponding 4 magnetic fields vectors. The distance between the spacecraft and the magnetic field are measured with uncertainties Δd and ΔB . The effect of these uncertainties is simulated by perturbing these vectors by adding a white noise on

each components, with an amplitude $\Delta B/B=1\%$ for instance. Then the current density is estimated by the contour integral method or the barycentric method (Ref. 1 and 3), and the relative error $\Delta J/J$ is deduced from the difference between the estimated value J_e and the (true) value J_m given by the model: $\Delta J/J=(J_e-J_m)/J_m$.

2.5. Relation between $\Delta J/J$ and geometric quality factors

To investigate the relation between $\Delta J/J$ and the various geometrical quality factors, many current structure crossing events have been studied, corresponding to a large number of Cluster tetrahedra (1000 or 2000), for each quality factor; statistical plots are produced such as those shown in Fig. 1. Each point corresponds to one configuration of the Cluster tetrahedron, and the relative error $\Delta J/J$ is plotted versus the value of the corresponding quality factor. Various symbols are used to identify the kind of the tetrahedron selected in the reservoir: perfectly regular or almost regular (square), almost planar (triangle), almost linear (diamond). The geometrical quality factor used for this example is deduced from the axes of the ellipsoid which fit best with the constellation geometry (Schoenmaekers, Ref. 5). The respective semi major axes, semi middle axes and semi minor axes of the ellipsoid are noted a , b , and c , and the value of this criterium, named $Q19$ in the list of studied criteria, is computed as $Q19=[(a+b+c)/3a - 1/3]*3/2$. The expression of $Q19$ is chosen to remain between 0 and 1.

Figure 1 shows that this kind of geometrical criterion perfectly separates the different families of tetrahedra, and is therefore useful to give an idea about the geometrical shape of the tetrahedron (regular, pseudo/planar, pseudo/linear). Furthermore, since one has always $a>b>c$, there is a one-to-one correspondence between the range of value of this criterium and the shape of the corresponding tetrahedron: criterium 19 can be considered as a measure of the "dimension" of the tetrahedron; a regular tetrahedron gives a value between 0.66 and 1, a pseudo/planar a value between 0.33 and 0.66, and a very elongated (pseudo/linear) a value between 0 and 0.33. Nevertheless, this kind of criterion cannot be used as a quality factor for the estimate of the current density. To get a relative error on $\Delta J/J$ of less than 1%, one has to consider values of $Q19$ such as $Q19<0.55$, thereby selecting only the almost regular and the perfectly regular tetrahedra, and to reject all other configurations. Given that the shape of the Cluster tetrahedron evolves along Cluster trajectory and that it is only regular at two points per orbit, this criterion would lead to reject most of the measurements.

In a preceding work (Ref. 3) we have shown that there exists two classes of geometric criteria: The first type of criteria is very useful to get an easy determination of the shape of the tetrahedron, as discussed above. The second type of criterion is a good indicator of the accuracy in the determination of J . For example, $Q10$ is computed as $Q10=C*(V/V_{sphe})^{1/3}$ where V is the volume of the tetrahedron, V_{sphe} is the volume of the sphere defined by the 4 points corresponding to the summits of the

tetrahedron, and C is a constant value to normalize the parameter between 0 and 1. This result is interesting, because if one sets the accuracy of the estimate of the current density to 1%, all tetrahedra with $Q10>0.4$ are selected (see Fig. 2). Of course, regular tetrahedra are selected, but some of the pseudo/planar and pseudo/linear tetrahedra are also selected, at variance with the criterium $Q19$ discussed above. Thus criterion $Q10$ allows to select irregular tetrahedra, which allows a reasonably good estimate of the current density.

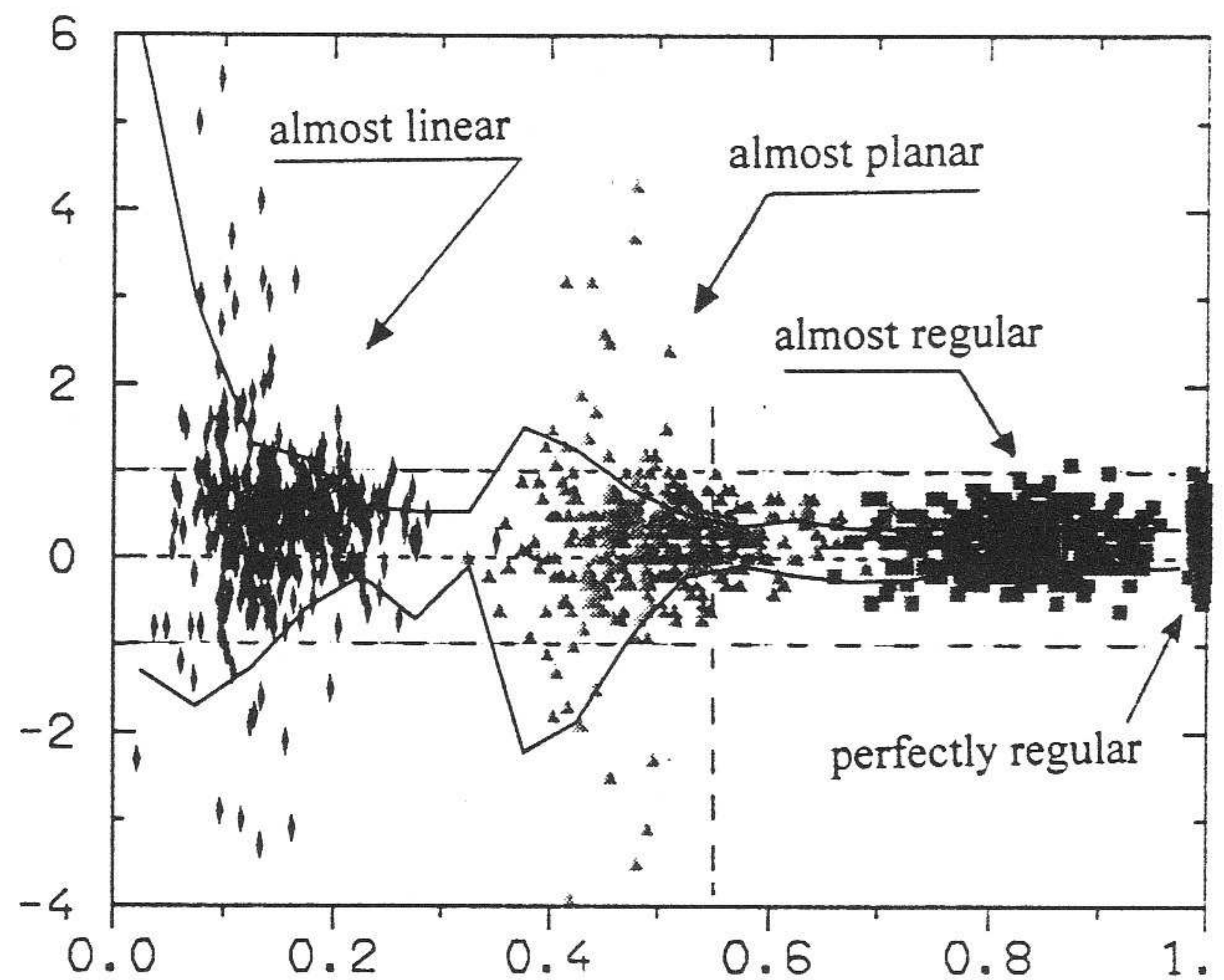


Figure 1. Example of geometric parameters related to the geometric shape of the tetrahedron. For 2000 tetrahedra, the $\Delta J/J$ relative error is plotted versus value of $Q19$ geometrical quality factor. $Q19=[(a+b+c)/3a - 1/3]*3/2$. (see text). Four populations of tetrahedra are used: (i) perfectly regular or (ii) almost regular (square), (iii) almost planar (triangle), (iiii) almost linear (diamond).

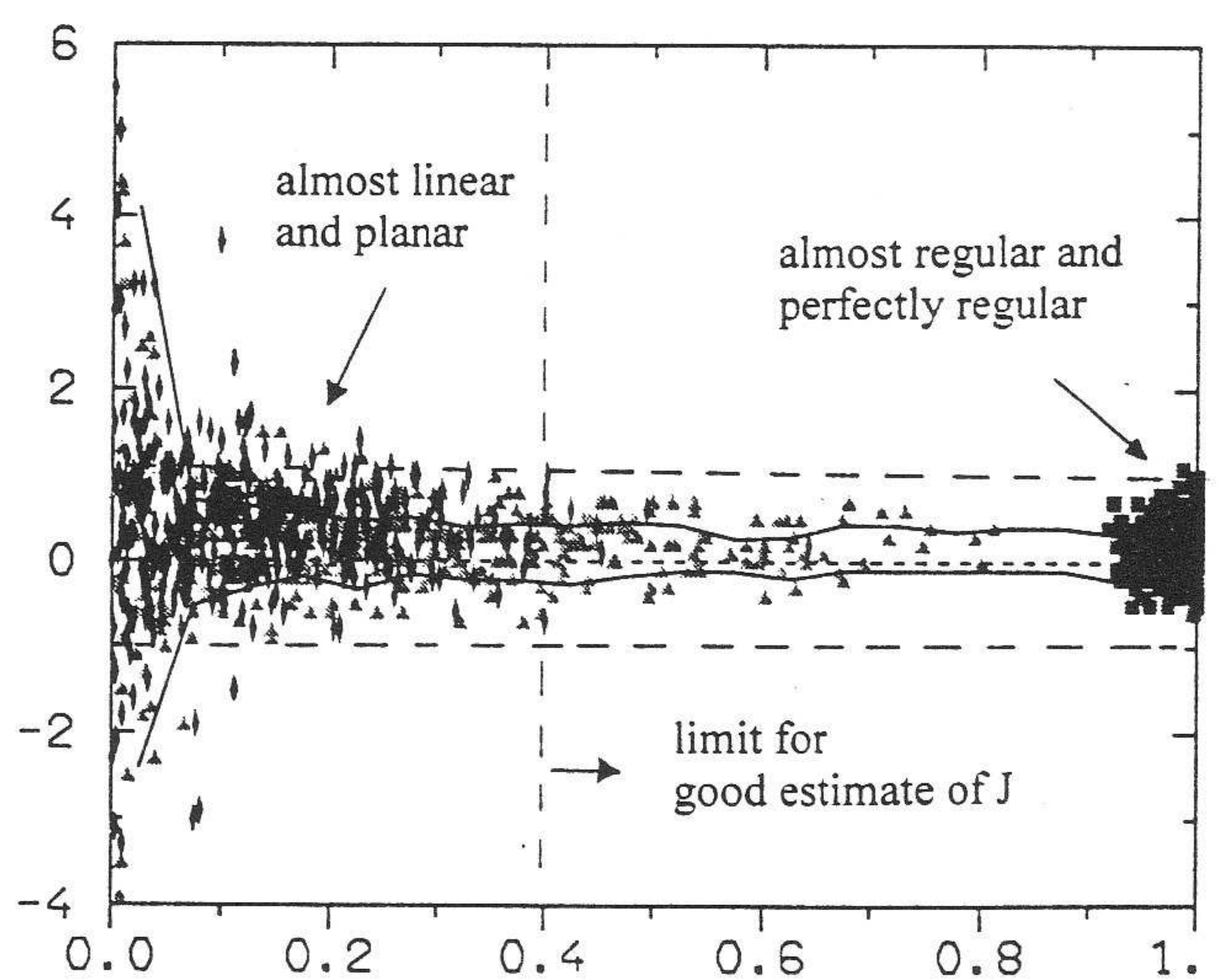


Figure 2. Example of geometric quality factor giving information about the accuracy of the estimate of the current density. $Q10=C*(V/V_{sphe})^{1/3}$ (see text). Simulation is made in the same conditions as for Fig. 1.

3. APPLICATION TO THE CLUSTER ORBIT

3.1. Cluster orbit and magnetic field model

The Cluster orbit was provided by ESA (Ref. 6). Two portions of this orbit are used: first a period where the apogee is in the tail at about $20 R_E$ (December 1996), and a second period where the perigee is in the tail, near the Earth at about $4 R_E$ (June 1996). The magnetic field model used to simulate the fluxgate magnetometer data is the Tsyganenko 1987 magnetic field model (Ref. 7).

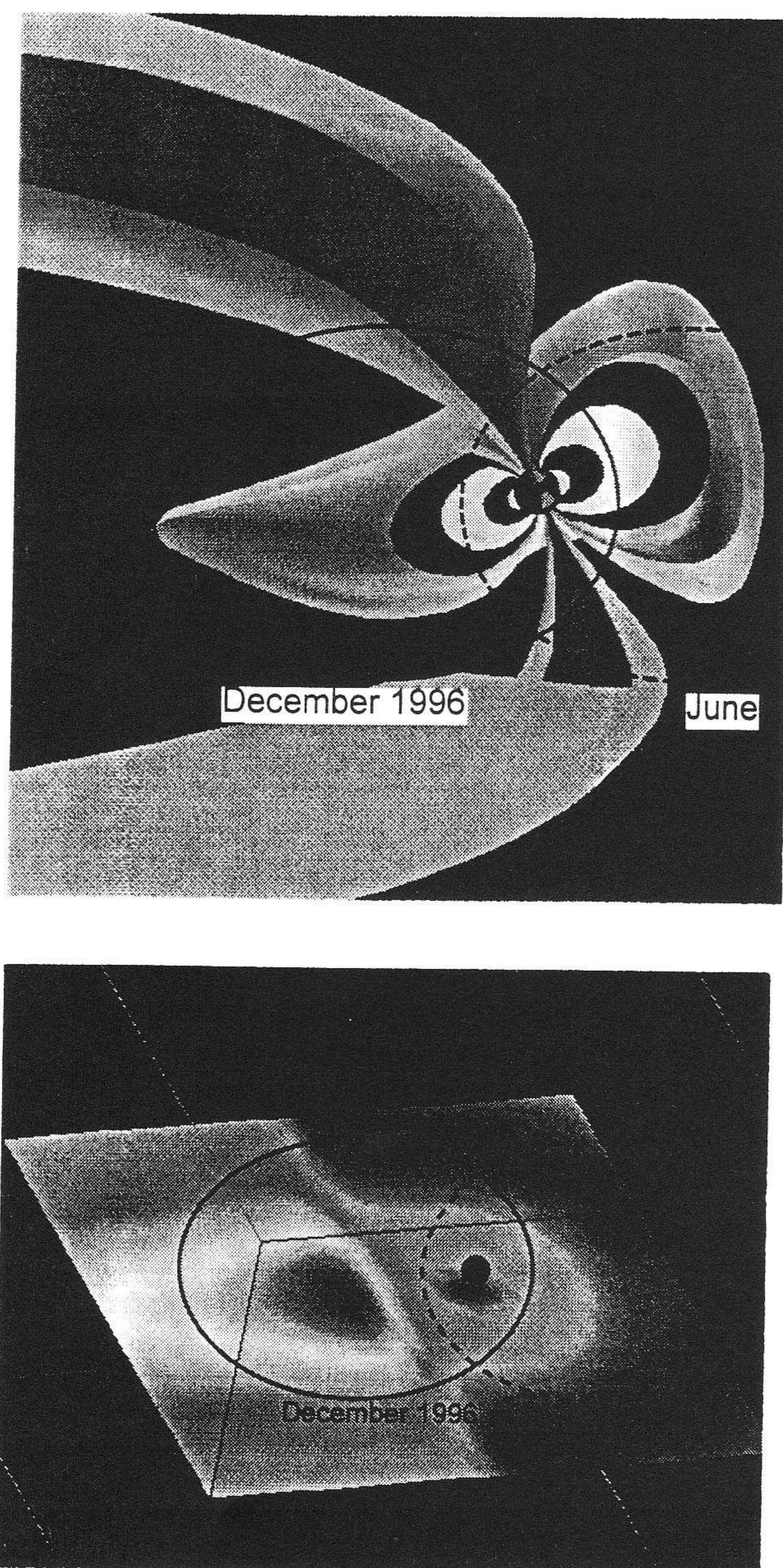


Figure 3. Top: 3-D representation of the magnetic shell, using the Tsyganenko 1987 magnetic field model. Bottom: current density in the equatorial plane deduced from the magnetic field model and computed from the finite differences with arbitrarily small steps to minimize the effect of the linear interpolation. On both figures, the 2 chosen Cluster orbits are plotted.

Fig. 3-a shows a 3-D representation of the magnetic shells, and Fig. 3-b shows the current density in the equatorial plane, computed from the finite differences with arbitrarily small steps to minimize the effect of the linear interpolation. For the two figures, (Figs. 3a, b) the corresponding portions of the orbit are plotted. It can be seen from Fig. 3-a that Cluster orbit is such that it does not cross the maximum of the tail current; near the apogee, in December, however, the current density along the orbit is large but the gradient in the magnetic field is weak, whereas in June, Cluster crosses the current sheet close to the Earth, in a region where the current density is small and the gradient in the magnetic field is large.

3.2. Cluster configuration and corresponding geometrical quality factors

Fig. 4 gives information about Cluster configuration for the June 1996 orbit. At the top of the figure, the configuration is given by 3 parameters: (i) the distance from the Earth, (ii) the 6 inter-spacecraft distances, showing the two times along the orbit where the 6 distances are equal (where the tetrahedron is regular), and (iii) the volume of the tetrahedron, which is null two times along the orbit, corresponding to a degenerate tetrahedron, plane or linear.

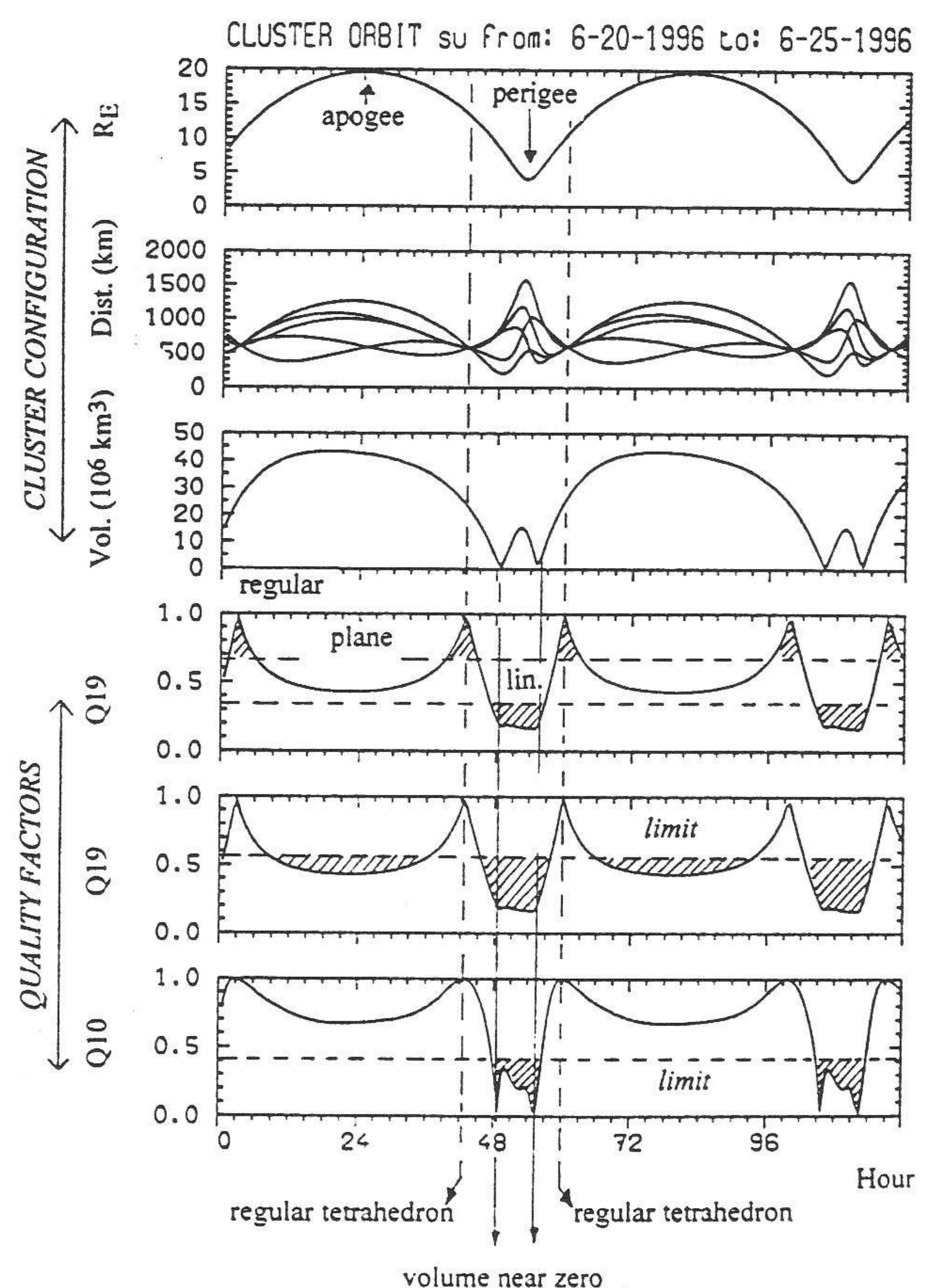


Figure 4. Cluster configuration (top) and geometric quality factors (bottom) during the June 1996 Cluster orbit.

At the bottom of the figure, the Q19 and Q10 geometrical quality factors have been plotted. Q19 is plotted twice. First time to show the shape of the tetrahedron; when the 6 inter-spacecraft distances are equal, the tetrahedron is regular and of course Q19 has a value equal to 1. But the Q19 value also gives more interesting information: during a major part of the orbit, the tetrahedron is rather flat ($0.4 < Q19 < 0.65$), and during a short part of the orbit, it is very elongated (almost linear, ($Q19=0.2$)). The second plot of Q19 shows that one cannot use it to characterize the quality of the measurement: the value of $Q19 > 0.55$ deduced from Fig. 1 would lead to the selection of a very small fraction of the orbit, corresponding to regular or almost regular tetrahedra. To characterize the quality of the current density measurement, the Q10 quality factor is more appropriate: first, we have seen that this criterion is easily related to the accuracy of the estimate of the current density, and second, one can see that if one selects the region $Q10 < 0.4$ only a small portion of the orbit is rejected. This corresponds to the period where the tetrahedron is degenerated or strongly elongated; therefore it is not surprising to get a poor accuracy in the determination of J.

4. RESULTS

4.1. Long tail crossing

Fig. 5 shows the detection of the far tail current sheet crossing in December 1996, at a time corresponding to the apogee of Cluster at about $20 R_E$. The magnetic field components are represented at the top of the figure, in polar coordinates (GSE). The minimum value of the modulus of B corresponds to both the apogee and the center of the current sheet. A reversal in the direction of the tail magnetic field corresponds to a change from 0 to 180° in the azimuthal component. The modules of $\text{curl}\mathbf{B}$, $\text{div}\mathbf{B}$, $\text{grad}\mathbf{B}$ are plotted in the middle of the figure. The theoretical value of $\text{curl}\mathbf{B}$ is plotted together with the estimated one, so as to estimate the accuracy in the determination of J.

The geometrical quality factors are plotted in the bottom of the figure. The Q19 criterion, considered as representative of the geometrical shape, shows that the tetrahedron is generally flat. If one considers Q19 as a quality factor, its value is below the value 0.55 quoted above; therefore the quality of measurement should be poor, which is not the case, the estimate of $\text{curl}\mathbf{B}$ being very close to the exact value (see middle panel of Fig. 5). This confirms that geometric criteria such as Q19 are related to the geometric shape of the tetrahedron, but does not allow an assessment of the quality of the measurement. On the other hand, the Q10 criterion discussed above gives good results: during the whole period where the current sheet crossing is detected with a good accuracy, the Q10 criterion gives a value greater than the value 0.4 quoted above. Therefore, the Q10 criterion is a good estimator of the accuracy in the estimate of the current density.

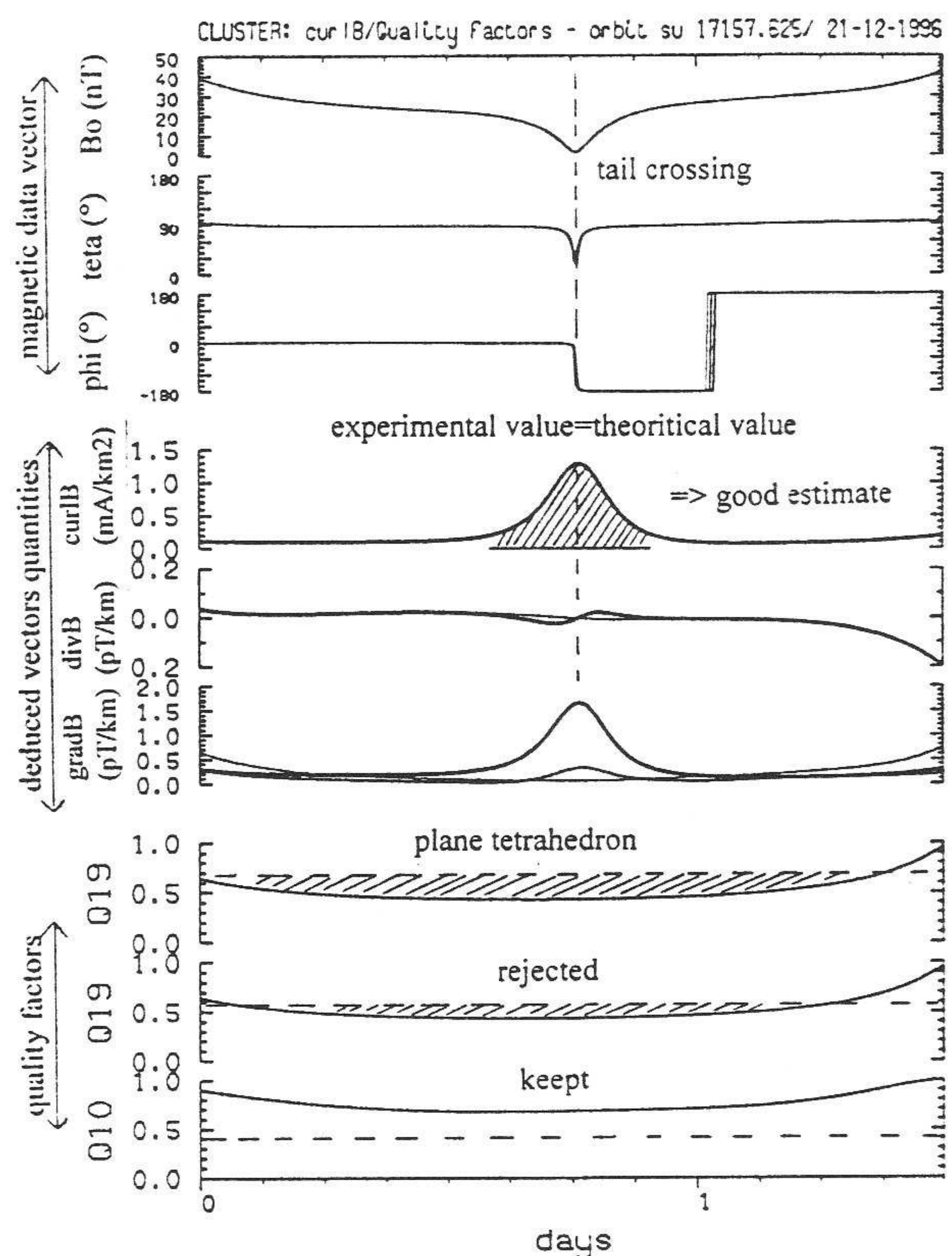


Figure 5. Validity of the estimate of the current density for December 1996 Cluster orbit, showing a good estimate of J. Top: magnetic field data in polar coordinates in GSE system. Middle: vectorial parameters of $\mathbf{B}$ ($\text{curl}\mathbf{B}$, $\text{div}\mathbf{B}$, $\text{grad}\mathbf{B}$). Bottom: geometric quality factors Q19 and Q10.

4.2. Short tail crossing

Fig. 6 shows the detection of the current sheet crossing in June 1996, corresponding to the perigee of the Cluster orbit at about $4 R_E$. In this region, the magnetic field is very strong and close to the dipole model, as can be seen in Fig. 3. In this region, the theoretical value of J is very weak, but the estimate of J by the contour integral method or barycentric coordinates method gives much higher values than the real ones, thus the estimate of the current density is very poor. The examination of the quality factor Q19 shows that the tetrahedron is rather linear during this time period, which explains why the estimate is poor. The Q10 criterion yields a value below 0.4 and therefore leads to the conclusion that the estimate of the current density is not good enough to be kept. Here again, the Q10 criterion is good at selecting periods where J can be measured within a reasonable accuracy.

5. CONCLUSION

The Tsyganenko magnetic field model has been used to simulate magnetic field measurements along the trajectories of the 4 Cluster spacecraft. The estimate of J was made from the barycentric coordinate method or contour integral method, when Cluster spacecraft cross the tail current sheet at small and large distances. The main problem is to know whether the estimate of J is significant or not. From the two examples studied, when Cluster is in the far tail (December 1996 orbit, apogee in the tail at $\sim 20 R_E$), the estimate of J is very good. Conversely, for short tail crossings (June 1996 orbit, perigee in the tail at $\sim 4 R_E$), the estimate is not correct and can reach more than 10 times the real value. Two types of geometrical criteria have been applied to this simulated data set: (i) the Q10 criterion is a good indicator of the accuracy of the estimate of J ; it can be qualified as quality factor, (ii) the Q19 criterion is not well related to the accuracy of the estimate of J but it is very useful in that it gives a quick answer as to what is the shape of the Cluster tetrahedron; it is a good indicator of the dimension of the tetrahedron (Ref. 9). It is suggested to plot on a regular basis these two criteria together with Cluster data, to get a quick indication about the shape of the tetrahedron and the accuracy of the estimate of the current density.

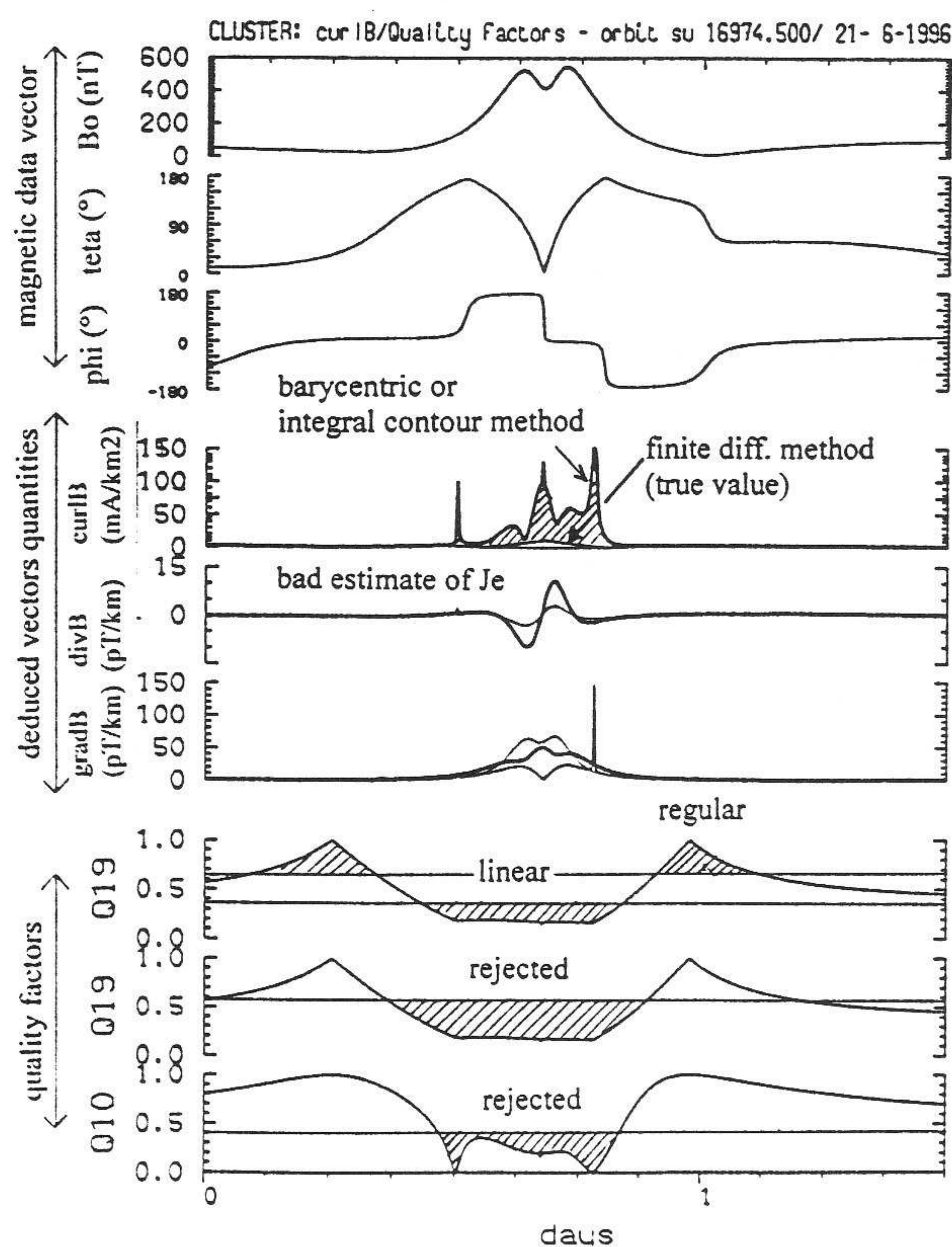


Figure 6. Validity of the estimate of the current density for June 1996 Cluster orbit, showing a poor estimate of J . Top: magnetic field data in polar coordinate in GSE system. Middle: vectorial parameters deduced from the measurement of B ($\text{curl}B$, $\text{div}B$, $\text{grad}B$). Bottom: geometric quality factors Q19 and Q10.

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