SIMULATED DAILY SUMMARIES OF CLUSTER FOUR-POINT MAGNETIC FIELD MEASUREMENTS

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ABSTRACT

Coordinated measurements to be carried out onboard the 4 Cluster spacecraft will give access to new parameters that could not be deduced from single spacecraft measurements. In the present paper, we compare various methods proposed to estimate curl **B** (and therefore the current density J), div **B** and grad **B**, and plot these estimates along the Cluster trajectory. These plots can be used as simulated daily summaries and allow a quick inspection of where, along the orbit, the estimate of Curl **B** and other differential quantities is accurate. It is shown that the estimated value of div **B** along the Cluster trajectory cannot be easily related to the error in the estimate of the current density.

1. INTRODUCTION

Cluster will provide four sets of time series of the magnetic field vectors along the orbits of the 4 spacecraft, with, for the first time, spatial and temporal resolution. It has been shown (Ref. 1, 2, 3) that the validity of the estimate of the current density J depends upon several factors, including (i) the accuracy of the fluxgate magnetometer, (ii) the knowledge of the distance between the spacecraft, (iii) the validity of the linear interpolation between measurements carried out at different locations, and (iv) the (geometrical) shape of the tetrahedron formed by the 4 spacecraft. We assume here that the measurement of the vector magnetic field aboard the 4 satellites and their position are known without any uncertainty. Two methods are used: contour integrals and barycentric coordinates to estimate J. The two methods are based upon the same data set, namely the 4 x 3 magnetic field vector measurements aboard the 4 spacecraft.

The Barycentric Coordinate method (BC) is independent of the geometry of the tetrahedron; it allows to get an estimate of the various differential quantities (curl **B**, div **B** and grad **B**) for an arbitrary geometry. The Contour Integral method (CI) is also very efficient but can only be used to estimate curl **B**. It is important to the success of the Cluster mission to know where, along the orbit, these methods provide accurate estimate of the current density. To simulate the measurements of the vector magnetic field at the locations of the 4 Cluster s/c, we use the Tsyganenko 1987 model (Ref. 4) and the evolution of the Cluster tetrahedron is computed from a program delivered by ESOC (Ref. 5).

The simulated measurements are taken either in the far plasmasheet (long tail crossings) at ~ 19.6 Earth Radii (R_E) or in the near plasmasheet (short tail crossings) at ~ 4 R_E. The current density deduced from the simulated measurements along the orbit are compared with "theoretical" values obtained from virtual spacecraft with arbitrarily small interspacecraft distance and forming a perfectly regular tetrahedron. This comparison allows a real time estimate of the effect of the error.

2. METHOD

2.1. Variations of the Cluster geometry

The evolution along the (mean) orbit of the shape of the tetrahedron formed by the 4 Cluster spacecraft has been studied by ESOC for each orbit. Here, we use a subset of these orbits displayed in Fig. 1, together with the different geometric parameters in GSE system.

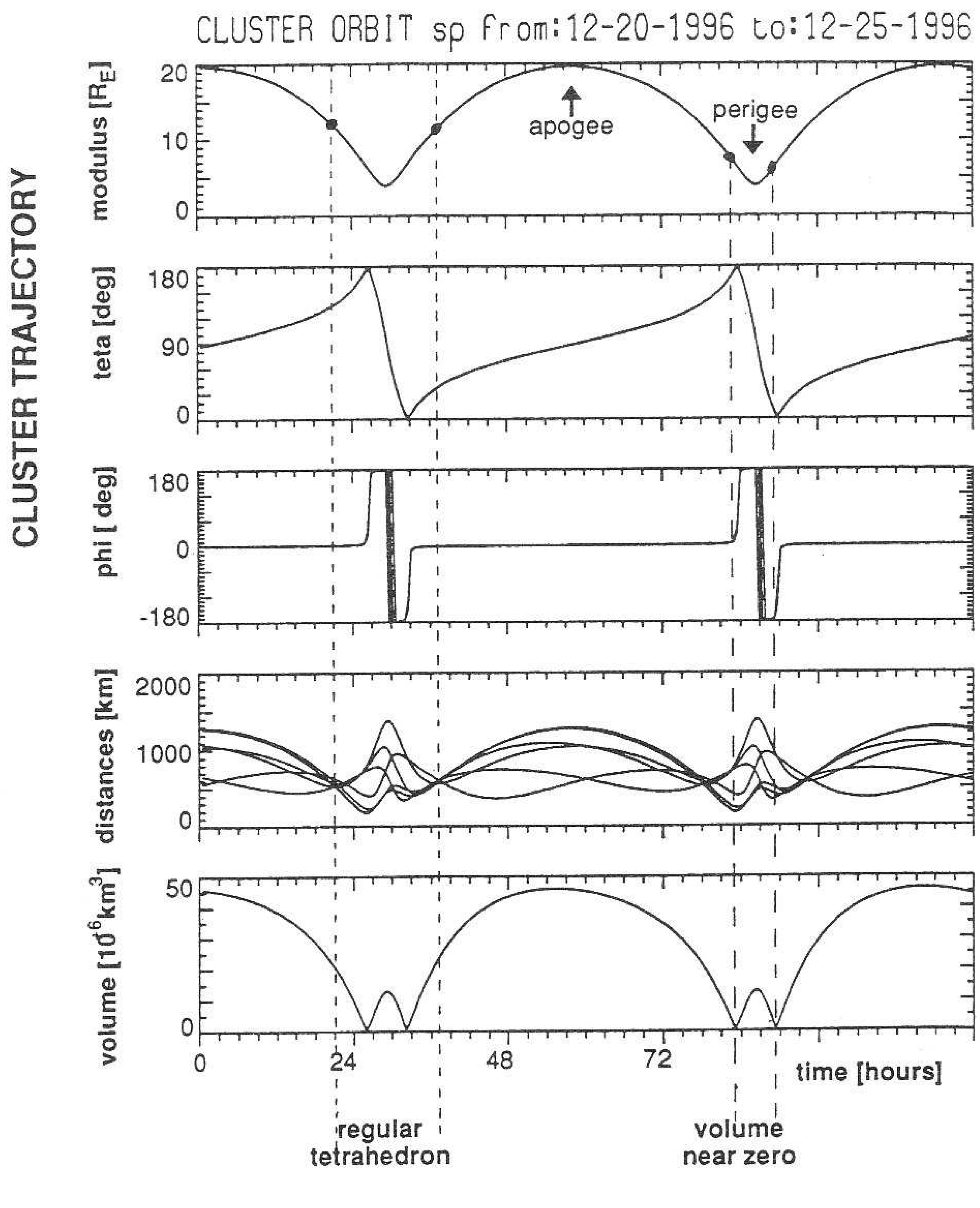

Fig. 1. Variations of the Cluster geometry during 5 days (GSE system). From top to bottom: positions of the spacecraft (spherical coordinates), interspacecraft distances and volume of the tetrahedron formed by the 4 Cluster s/c.

Proceedings of the Cluster Workshop on Physical Measurements and Mission Oriented Theory, Toulouse, France, 16–17 November 1994 (ESA SP-371, June 1995)

The four spacecraft positions, plotted in spherical coordinates r, θ and ϕ during the whole orbit, are consistent with a polar orbit (ϕ remains constant) with an apogee at about 20 R_E and a perigee at about 4 R_E. The interspacecraft distances plotted in km, show that around the perigee, the variations of the Cluster geometry are important between the two particular points where the tetrahedron is regular. The volume of the tetrahedron becomes null very close to the perigee, which means that the 4 Cluster satellites are more or less located in a plane or highly elongated.

2.2 Parameters deduced from field measurements

We study the quantities derived from the four-point measurements of **B** and obtained via two different methods: the Contour Integrals (CI) and the Barycentric Coordinates (BC) ones. These two methods use four points measurements of **B** at the four vertices of the Cluster tetrahedron, and rely upon the assumption that the variation of **B** is linear between two measurement points (2 s/c).

The CI method calculates the integral of **B.dl** around the contour defined by triangles limiting each face of the tetrahedron (Ref. 2). Three vectors **J** are determined through three surfaces defined by Cluster and converted into a regular coordinates system. Then curl **B** can be obtained directly with the CI method.

The BC method (Ref. 6) uses the barycenter coordinates of the tetrahedron. Values of grad **B** are first calculated and then div **B** and curl **B** can be directly obtained from grad **B**.

The results obtained from these two methods can be compared to theoretical values obtained by taking a regular and orthogonal tetrahedron along the whole orbit, and applying a finite difference scheme at the barycenter of this reference tetrahedron, with an arbitrarily small distance. The values of **B** are given at all points in space by the Tsyganenko model. This method, called "Finite Difference" (FD) in the paper, does not correspond to a realistic situation and can only be used with a theoretical field model, which gives a value of **B** everywhere, thereby allowing to keep the distance small enough to eliminate the error related to the linear interpolation.

The comparison between this accurate theoretical estimate (FD method) and the one which is obtained with a realistic Cluster tetrahedron geometry (CI and BC) allows a determination of the error associated with the geometry of the Cluster tetrahedron.

3. RESULTS

3.1. Long tail crossings

We first calculate the magnetic parameters derived from four-point measurements on December 21, 1996, when along the nominal orbit Cluster crosses the tail at large distances (~ 20 R_E). Fig. 2. shows the corresponding trajectory of the Cluster spacecraft and the currents calculated from the Tsyganenko model. The region where the current density is the largest is located

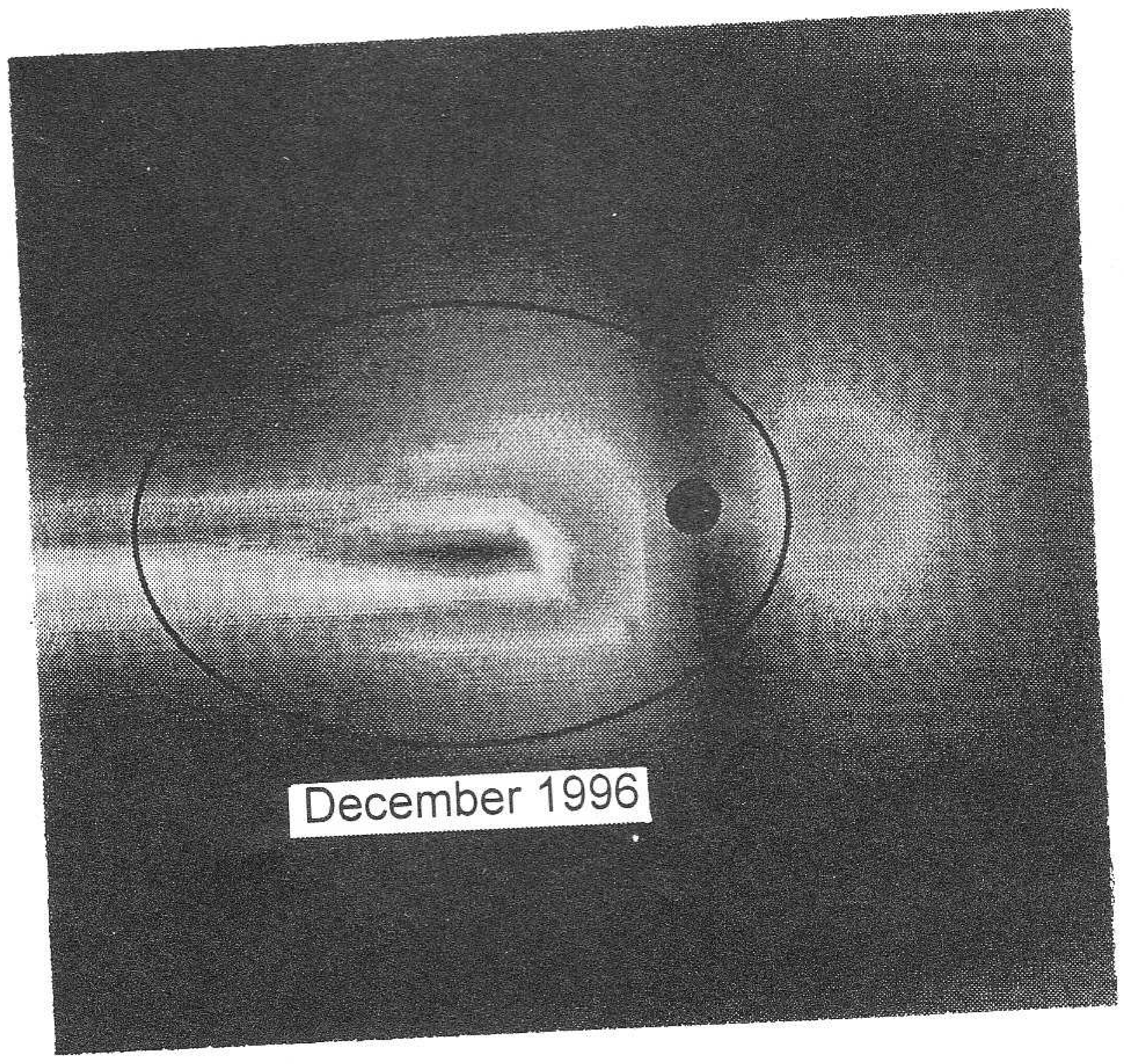

Fig. 2. Map of curl B calculated with the Tsyganenko model (GSE system) in a meridian plane. Cluster trajectory crosses the far tail near the apogee, in December 1996.

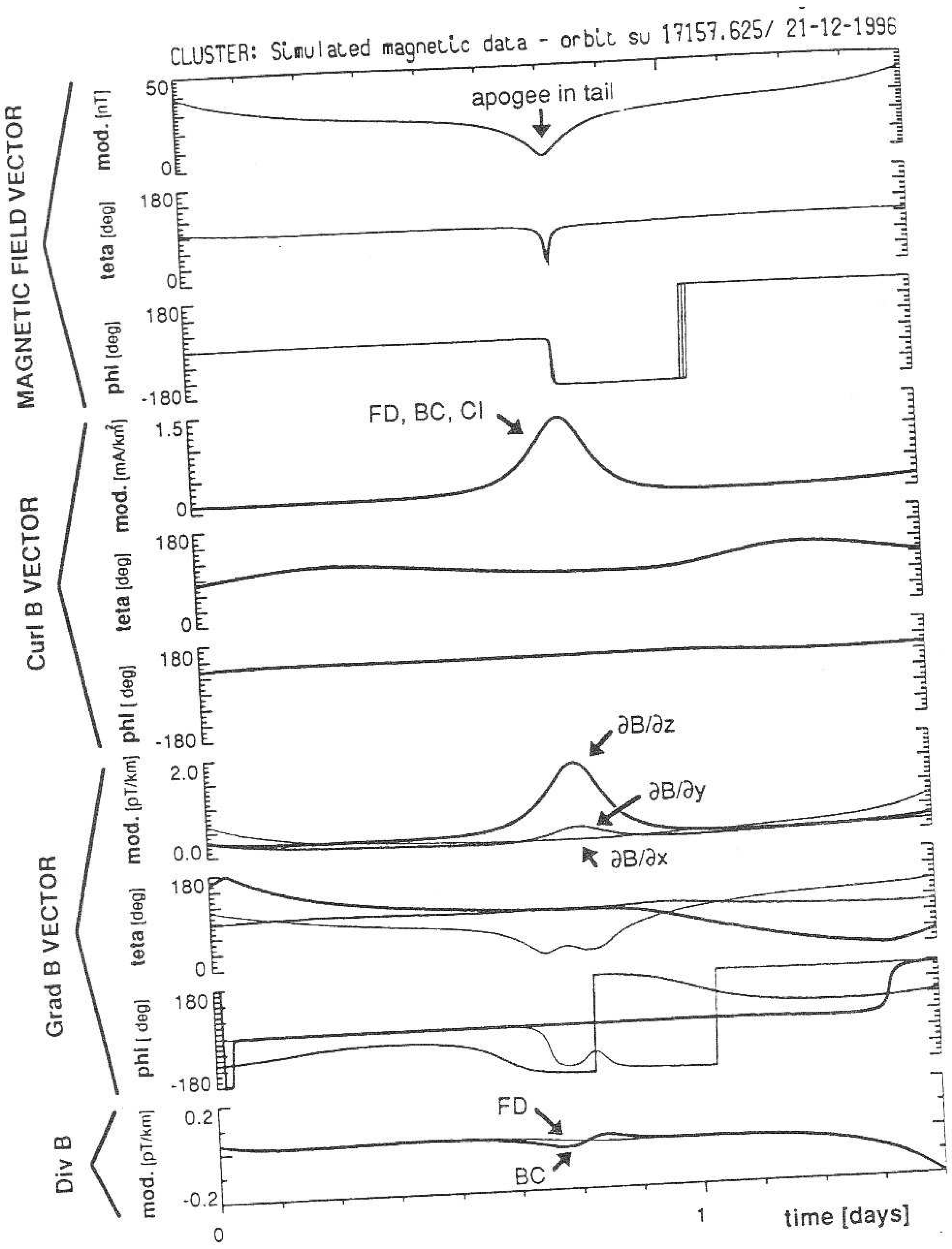

Fig. 3. Simulated magnetic daily summaries in the GSE coordinates for December 21, 1996. From top to Bottom: B, curl B, grad B and modulus of div B in polar coordinates.

relatively close to the Earth (~ 8 to 10 R_E) and is not crossed by the orbit, but Fig. 2 shows that near the apogee, at ~ 20 R_E, the current density is still quite large.

The magnetic vectorial parameters deduced from CI and BC methods are plotted in Fig. 3, as a kind of magnetic daily summaries, in the GSE system.
Modulus of **B** is minimum in the far tail since the apogee of the orbit, corresponds to a maximum distance from the Earth and a mimimum value of **B** at the center of the tail current sheet. The polar angle θ varies from 90° outside the tail, where the field lines are opened, to 0° inside the tail because Cluster encounters closed field lines. For a polar orbit, the azymutal angle ϕ changes abruptly from 0° to 180°.
First, Fig. 3. shows that the CI and BC methods give exactly the same results with same data and hypotheses, second, the two methods give about the same results as the theoretical one (FD) for curl **B**, thereby proving that in this case, close to the apogee, the estimate of curl **B** with a realistic four-point measurement method is good enough.
The BC method also permits to calculate the following components of the gradients of **B**: $\partial \mathbf{B}/\partial x$, $\partial \mathbf{B}/\partial y$ and $\partial \mathbf{B}/\partial z$. The orientation of the vector $\partial \mathbf{B}/\partial z$ (θ of 90° and ϕ of 0°, Fig. 3.) indicates that the term $\partial Bx/\partial z$ (i.e. the variation of Bx along the z direction) is the most important, as expected. The values of div **B**, obtained from the BC method or the theoretical FD method, are almost the same. The intensity of div **B**, 0.02 pT/km, is much lower than the intensity of curl **B** , 1.6 pT/km (1.6 pT/km $\approx$ 1.3 mA/km^2). Thus the value of div **B** could be related to the error in the estimate of J, at least in this case.

3.2. Short tail crossings

In a second case, the simulated experiment takes place in June 21, 1996, as shown in Fig. 4., while the current sheet is crossed near the perigee of Cluster (about 4 R_E).

The parameters deduced from magnetic field measurements are plotted in Fig. 5, for the June 21, 1996 crossing.
The intensity of **B** is, as expected, 10 times larger than in the previous case, since Cluster is closer to the Earth. The results obtained by the CI and BC methods for the estimate of curl **B** are the same but they differ much from the theoretical values given by the FD method.
Fig. 5. shows clearly that, in this second case (short-tail crossings), the four point measurements methods provide erroneous estimates of curl **B** when the measurements aboard the 4 spacecraft are simulated. The evolution along this part of the orbit of div **B** is about the same as that of curl **B**; its amplitude is also similar (15 pT/km for div **B** and ~ 20/25 pT/km for curl **B**).

To help finding an explanation of these results, we present again the same results in Fig. 6 (curl **B** and div **B** for June 21, 1996) but with an enlarged scale.

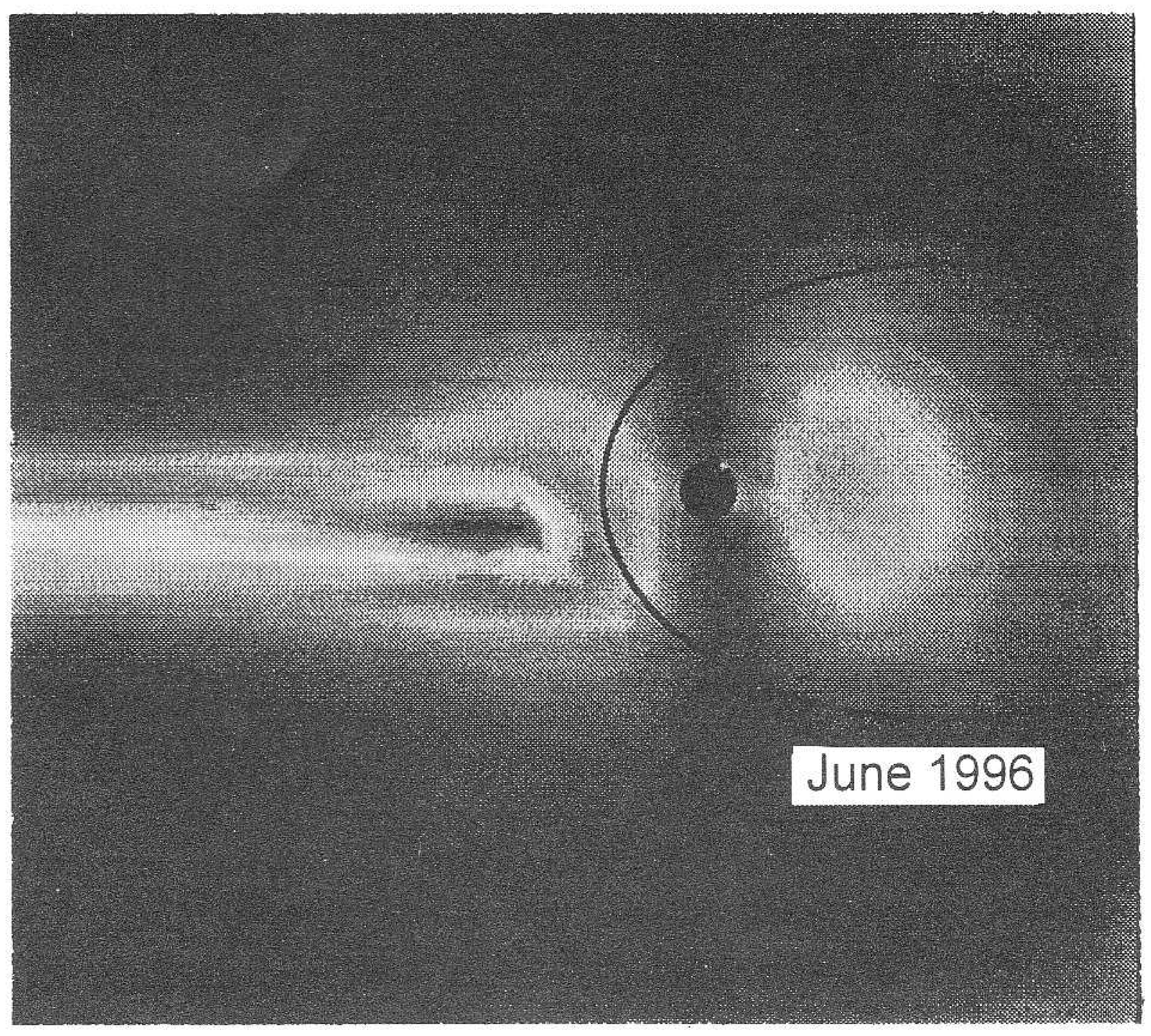

*Fig. 4. Map of curl **B** calculated with the Tsyganenko model (GSE system) in a meridian plane, as Cluster trajectory crosses the tail near the perigee, in June 1996.*

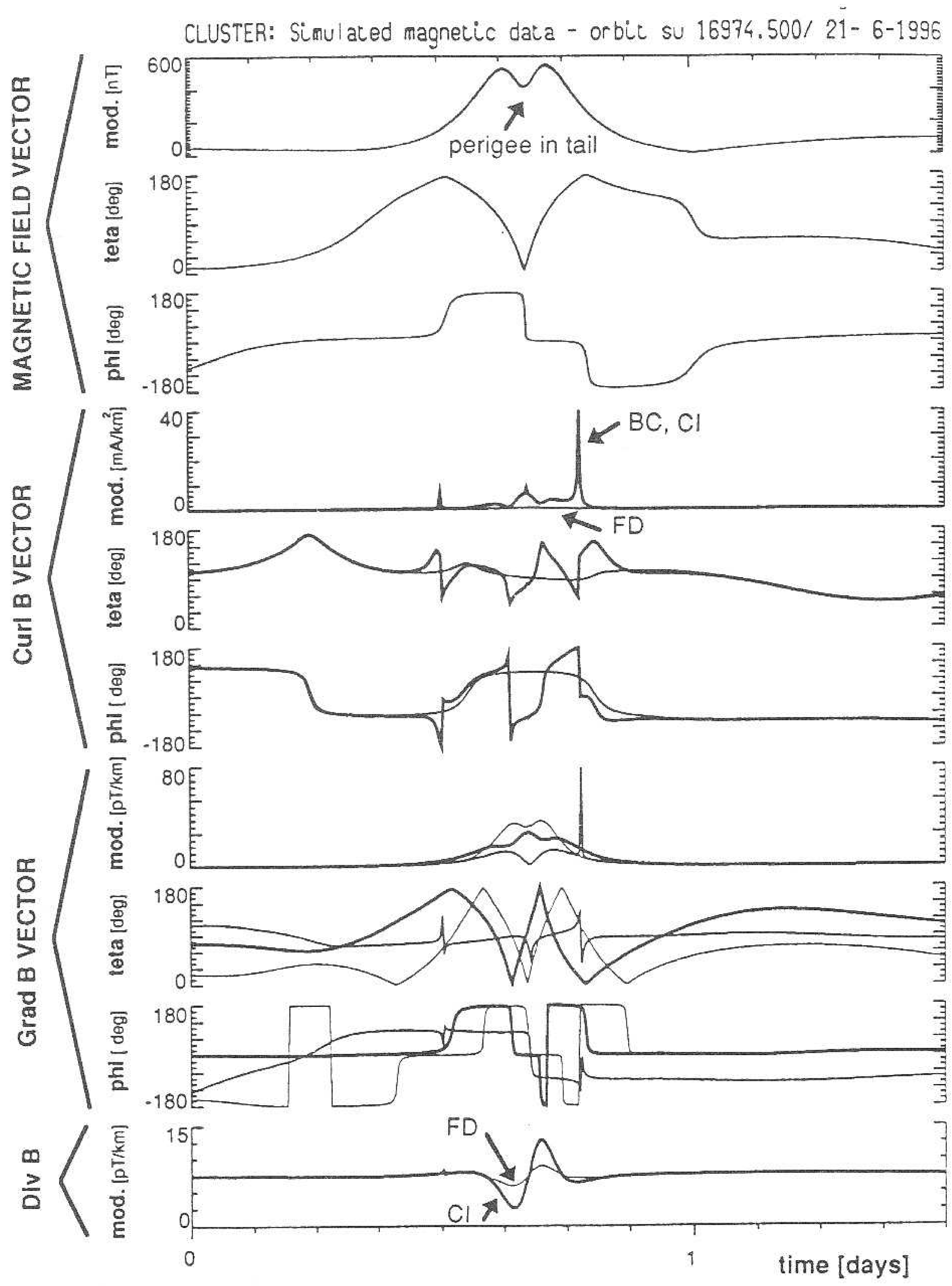

*Fig. 5. Daily summaries (GSE system) deduced from 4 point-magnetic field measurements, 21 June 1996 (short tail crossing). From top to bottom: **B**, curl **B**, grad **B** and modulus of div **B** in polar coordinates.*

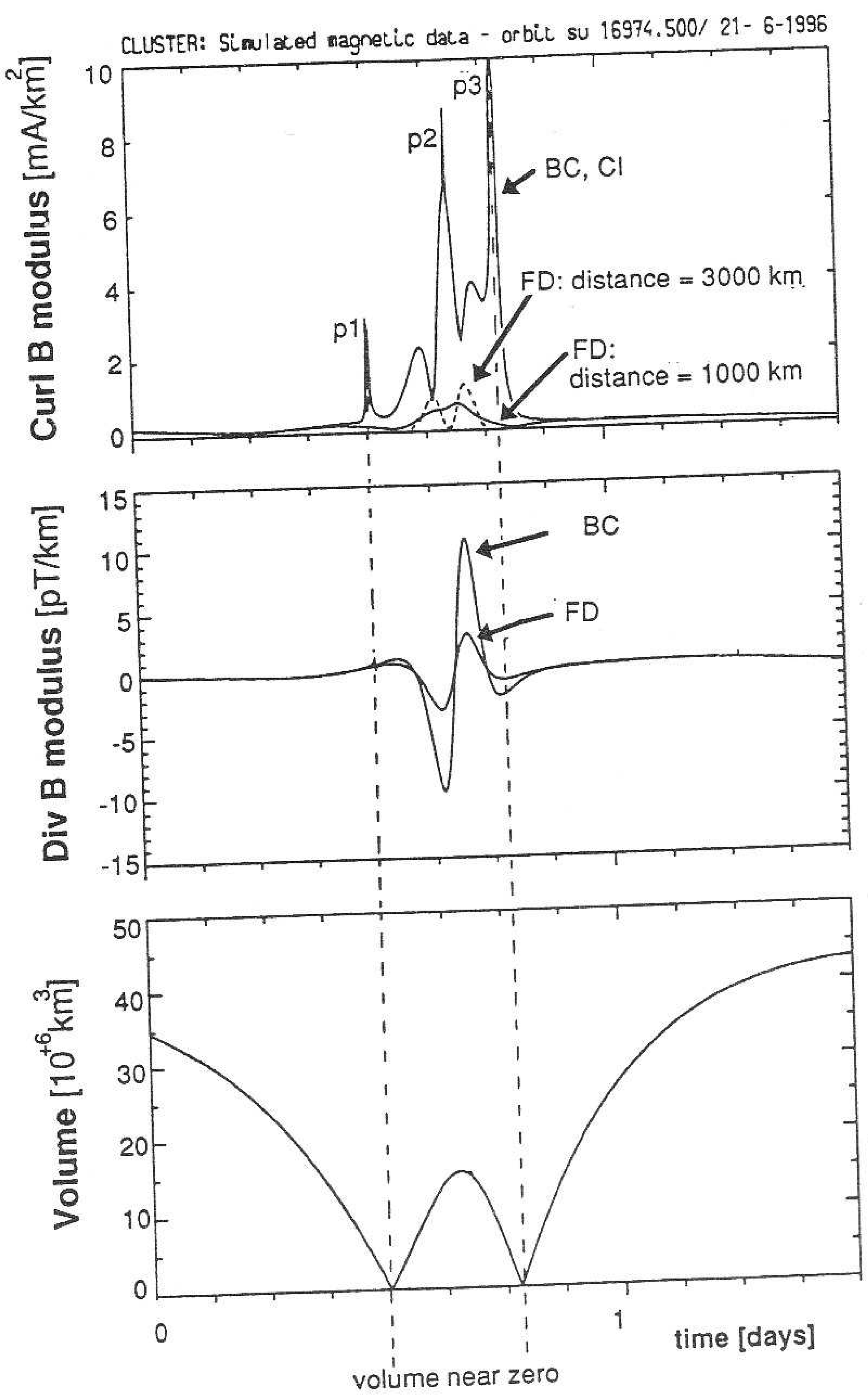

*Fig. 6. Magnetic daily summaries, same as Fig. 6, but with a larger scale. From top to bottom: modulus of curl **B**, div **B**, volume of the tetrahedron.*

The variation of the Cluster geometry is given during the same time period by the volume of the tetrahedron, plotted at the bottom of Fig. 6. Two of the peaks (p1 and p3) in the estimate of curl **B** from the BC and CI methods are correlated with the volume of the tetrahedron; they correspond to the two points where the volume is near zero, thus it is not surprising that the estimate of curl **B** is very poor.
There are, however, other peaks that do not correspond to a null in the volume of the tetrahedron, for instance, the peak p2, just at the perigee. For this peak, the value of div **B** obtained from the BC and FD (theoretical) method are both close to zero. In spite of this, the estimate of curl **B** via the BC and CI methods (that mimic Cluster measurements) is not good. This indicates that div **B** is not always a good estimator of the accuracy in the determination of curl **B**.

As a further attempt to identify what causes these large errors in curl **B**, we have again used the FD method but this time, we have increased from 1000 to 3000 km the distance between the spacecraft, which allows to test whether the error is due to the linear interpolation. Then, not surprinsingly, we see from Fig. 6 that, with a regular tetrahedron but a large interspacecraft distance, the FD method also introduces errors. These errors, however, are smaller and are not found at the same location along the trajectory as the ones found with a realistic Cluster constellation (via the BC and CI methods). Since the average distance between the spacecraft is less than 3000 km on this part of the orbit, we are led to conclude that the dominant source of error, close to perigee is not the linear interpolation itself but, again, the geometry of the 4 spacecraft which is very degenerated during this time period, and therefore amplify by a large factor the error associated with the linear approximation.

4. CONCLUSION

The Tsyganenko 1987 magnetic field model has been used to benchmark the estimate of differential quantities derived from magnetic field vectors measured at the 4 Cluster spacecraft locations. The mean trajectory of the Cluster tetrahedron and its deformation along the orbit has been taken into account.
Since the tetrahedron is not usually regular, specific methods adapted to arbitrary geometry have to be developed. Curl **B** and therefore the current density can be calculated, for an arbitrary geometry, by Contour Integrals (CI) or by the Barycentric Coordinate (BC) methods. We have checked that the two methods give the same results but the BC method has the advantage that it can be used for estimating other differential quantities such as div **B**, grad **B**, etc..., in addition to curl **B**, and is more easily implemented in a computer.
In an attempt to determine where, along the orbit, the estimate of differential parameters is good enough, we have plotted curl **B**, grad **B** and div **B** deduced from a realistic model of the evolution of the Cluster tetrahedron along the nominal orbit. For long tail crossings, the estimate of the current density deduced from the magnetic field given by the Tsyganenko model is very accurate; measurements by BC and CI methods carried out aboard the model spacecraft give about the same value as an ideal tetrahedron (regular and with arbitrarily small distances between the points where **B** is measured). Conversely, for short tail crossings, the estimate via CI and BC of the current density is often very poor, i.e. differs by a large factor from the model value.
We have shown that there is no obvious relationship between the accuracy of the curl **B** estimate and the estimate of div **B**. The error associated with the linear interpolation between the spacecraft cannot alone explain why the estimate of J along the orbit is sometimes very poor. We have shown that the regions where the estimate of J is very poor correspond to a degenerate tetrahedron, i.e. regions where the volume of the tetrahedron goes to zero and/or the tetrahedron is very elongated (linear tetrahedron).
Thus, instead of using div **B** as a criterion for curl **B** estimates, it is more instructive to build geometric criteria based for instance on the calculation of the volume defined by the 4 s/c. This is done in a companion paper by Robert et al.

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